

Quantum correlations in a noisy neutron interferometer

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(Dated: April 26, 2013)

We investigate quantum coherences in the presence of noise by entangling the spin and path degrees of freedom of the output neutron beam from a noisy three-blade perfect crystal neutron interferometer. We find that in the presence of dephasing noise on the path system the entanglement of the output state reduces to zero, however the quantum discord remains non-zero for all noise values. Hence even in the presence of strong noise non-classical correlations persist between spin and path of the neutron beam. This indicates that measurements performed on the spin of the neutron beam will induce a disturbance on the path state. We experimentally demonstrate this disturbance by comparing the contrasts of the output beam with and without spin measurements of three neutron interferometers with varying noise strengths. This demonstrates that even in the presence of noise that suppresses all entanglement, a neutron interferometer still exhibits uniquely quantum behaviour.

A unique property of quantum systems is that when two quantum systems are allowed to interact they may exhibit types of correlations that cannot be explained classically. In the field of quantum information science, protocols harnessing these correlations can in principle exceed classical efficiencies for certain information processing tasks, for example factoring large prime numbers, and performing unstructured search [1]. The strongest type of bipartite quantum correlation is entanglement. Maximally-entangled systems allow for projective measurements of one to completely determine the outcomes for corresponding projective measurements on the other. However non-entangled quantum systems may still possess correlations that cannot be accounted for classically. In such cases measurement on one subsystem, while not determining the state of another, may still cause a disturbance to its state. Quantum discord (QD) was proposed by Ollivier and Zurek [2], and Henderson and Vedral [3] to characterize quantum correlations in a bipartite system. In effect, one may view QD as a measure of how much disturbance measurement of one subsystem of a bipartite quantum system can induce on the other. QD has received much interest in recent years, and for a review of the topic we refer the reader to Ref. [4].

Modern advances in neutron interferometry have allowed for precise tests of quantum mechanical phenomena. For example gravitationally induced quantum interference [5], coherent spinor rotation [6] and superposition [7], dynamical diffraction through a thick crystal [8], the Aharonov-Casher effect [9], violation of a Bell-like inequality [10], quantum contextually [11], ³He

incoherent scattering lengths [12], and the realization of a Decoherence-Free subspace [13]. In our case a neutron interferometer provides a clean system for considering quantum correlations between bipartite quantum systems as we are able to coherently control the spin and path-momentum degrees of freedom of a neutron beam, and manipulate the correlations between them. In addition, due to the high visibility of single neutron detectors and the low intensity of neutrons entering the interferometer, we are able to gather statistics from performing true projective measurements on single quantum systems. In the present paper we investigate the correlations between the spin and path degrees of freedom of the output beam from a noisy neutron interferometer by observing changes in the output beam intensity as a result of a projective measurement on the neutron spin.

Quantum discord is a non-symmetrical quantity which measures the difference between the quantum generalizations of two classically equivalent expressions for the *mutual information* of a bipartite quantum system. The mutual information of the state ρ_{AB} of a bipartite quantum system AB is given by

$$I(A : B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}) \quad (1)$$

where $S(\rho) = -\text{tr}(\rho \log \rho)$ is the von-Neumann entropy of the density matrix ρ , $\rho_A = \text{tr}_B(\rho_{AB})$ is the reduced density matrix on subsystem A , and similarly for ρ_B . An alternative expression for mutual information is formed by considering a quantum generalization of conditional entropy which accounts for possible measurement induced disturbances. Consider performing a measurement on subsystem A , this is most generally described by a positive operator valued measurement (POVM) E consisting of a set of measurement operators $\{E_a\}$ satisfying $E_a \geq 0$, $\sum_a E_a = \mathbb{I}$ [1]. Measurement outcome a will

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occur with probability $p_a = \text{tr}(E_a \rho_{AB})$, and the post-measurement state of subsystem B , conditioned on outcome a is given by $\rho_{B|a} = \text{tr}_A(E_a \rho_{AB})/p_a$. We may define a generalization of conditional entropy for a given POVM E as $S(\rho_{B|E}) = \sum_a p_a S(\rho_{B|a})$. This gives us an alternative expression for mutual information by maximizing over all possible POVMs:

$$J(B|A) = \max_E [S(\rho_B) - S(\rho_{B|E})] \quad (2)$$

Quantum discord is defined to be the difference between expressions (1) and (2):

$$\begin{aligned} D(B|A) &= I(A : B) - J(B|A) \\ &= \max_E [S(\rho_{B|E}) + S(\rho_A) - S(\rho_{AB})] \end{aligned} \quad (3)$$

Similarly one may define the quantum discord $D(A|B)$ where one optimizes over POVMs on subsystem B . In general one must optimize quantum discord over extremal rank-one POVMs, however it has been shown that for rank-two states orthogonal projective measurements are optimal [14].

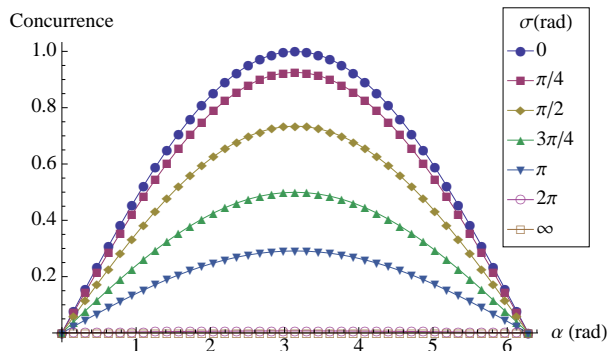


FIG. 1: Concurrence between spin and path degrees of freedom of output neutron beam as a function of the spin rotation angle α .

We now consider quantum correlations in the output state of a three-blade neutron interferometer, and will follow with a full description of the NI used to derive them. In our configuration systems A and B correspond to the spin, and path degrees of freedom of a neutron beam. By performing a controlled spin-rotation of angle α in one of the paths of the NI we may introduce entanglement between the spin and paths of the neutron beam. However, in a realistic NI there are many noise sources which introduce decoherence. In the present paper we consider the decoherence due to surface defects of the NI blades. This noise source introduces a random phase between the two interferometer paths which degrades phase information of the path system B .

To calculate the entanglement and quantum discord of the output beam of the noisy neutron interferometer we use *concurrence* as our measure of entanglement [15],

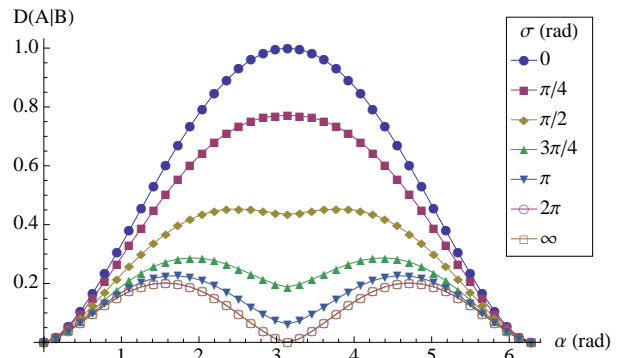


FIG. 2: Quantum discord $D(A|B)$ between spin and path degrees of freedom of output neutron beam as a function of the spin rotation angle α .

and since the output beam state is rank-two, we may numerically calculate the discord by optimizing over PVMs. We find that the entanglement between the spin and path systems goes to zero for all values of the spin rotation as the strength of the random phase noise increases. This removal of entanglement under dephasing noise for two-qubits has been previously noted and classed as *approaching* entanglement evolution [16]. The quantum discord in contrast remains non-zero for values not equal to an integer multiple of π radians. This is illustrated in Figs. 1 and 2 respectively. This shows that in the case of strong noise even though there is no entanglement between the spin and path of the neutron beam, the non-zero quantum discord $D(A|B)$ indicates that measurements performed on the neutron spin will induce a disturbance on the path state of the output neutron beam.

We will now briefly describe the mathematical model of an ideal three-blade neutron interferometer. The most common geometry for a neutron interferometer is a three-blade system machined from a perfect single crystal of silicon. This functions as a Mach-Zender interferometer on the longitudinal momentum of the neutron beam. We refer to this degree of freedom of the neutron beam as the *path* system. The neutron path can be viewed as a two level system which we may couple to the neutron spin to form a bipartite quantum system. In this context we may view the interferometer crystal as a quantum circuit acting as illustrated in Fig. 3.

The first (and third) NI blades act as Hadamard (H) gates on the neutron path by coherently splitting (and recombining) the neutron beam into two paths via Bragg scattering in the Laue geometry [17]. The second NI blade deflects the beam by swapping the path-momentum directions, which we model as a bit-flip (X) gate. In practice approximately half the intensity of the neutron beam is lost due to neutrons escaping the NI at the second blade, however for our description of the output beam we post-select on the neutrons which remain in the interferometer. Between the first and second NI blades we

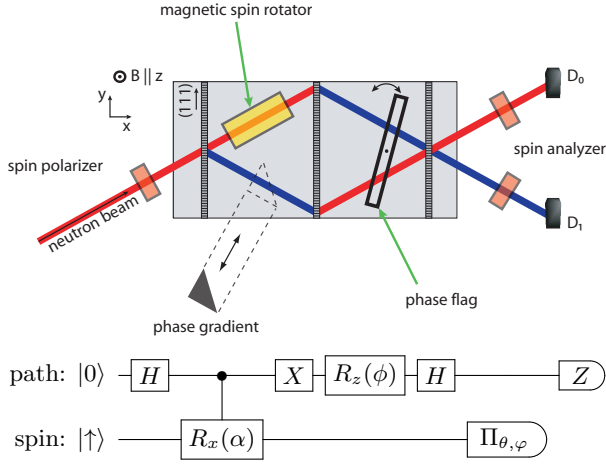


FIG. 3: Experimental setup for three blade neutron interferometer and the quantum circuit corresponding to the ideal model. H is a Hadarmard gate, $R_x(\alpha)$ is a rotation of the neutron spin of α radians about x -axis, X is a bit-flip, $R_z(\phi)$ is a phase shift of ϕ radians between the beam paths, Π is a projective measurement performed on the spin-state (spin-analyser), and Z is a projective measurement of the path intensities.

couple the spin and path degrees of freedom by selectively rotating the neutron spin in one path by an angle α . This acts a controlled- X rotation ($R_x(\alpha)$), with the spin and path as the target and control respectively. Finally, we measure the intensities of the output beams using two ^3He integrating detectors called D_0 and D_1 . This is effectively a Z -basis measurement on the path. By including spin-filters which selectively transmit neutrons with a preferred spin we may also perform post-selected spin measurements. This allows us to perform joint measurements on the spin and path of the neutron beam.

In a typical NI experiment a relative phase of ϕ is induced between the two paths by a phase flag between the second and third blades. This phase parameterizes the measured beam intensity by controlling the interference between the two beam paths recombined at the third blade. The intensity curves for each detector as a function of ϕ are referred to as *contrast curves*. The difference between the maximum and minimum intensity of the D_0 -detector as a function of a phase-flag rotation ϕ is called the *contrast* of the NI and is given by

$$C_0 = \frac{\max(D_0(\phi)) - \min(D_0(\phi))}{\max(D_0(\phi)) + \min(D_0(\phi))}. \quad (4)$$

In practice neutron interferometers cannot be machined perfectly. Surface imperfections in the blades of the interferometer crystal induce a random phase between the two interferometer beam paths. This results in reduced contrast of the beam intensity when averaged over the beam distribution. To include the effect of phase noise in our model we average over output states after introducing a random phase between paths. This random

phase is assumed to be normally distributed with mean 0 and variance σ . If we examine the contrast as a function of the noise, we find that $C_0 = \exp(-\sigma^2/2)$.

As the standard deviation of the noise increases, the intensities at both detectors approach a constant value of $I_0 = I_1 = 1/2$, and hence the contrast reduces to zero. This would suggest that the random phase noise destroys all relative phase information, and hence coherence, between the two paths in the interferometer. However, as shown in Fig 2, there is a non-zero discord $D(A|B)$ between the spin and path subsystems. Hence if we implement a measurement on the spin system the output intensities of the path system must be affected. We implement the spin measurement by a spin-filter which acts to post-select the output spin-state of the neutron to be spin-down. If we assume that our incoming neutron beam is initially polarized in the state

$$\rho_{spin} = \frac{1+\epsilon}{2} |\uparrow\rangle\langle\uparrow| + \frac{1-\epsilon}{2} |\downarrow\rangle\langle\downarrow| \quad (5)$$

with respect to a uniform magnetic field in the z -direction, our spin-filtered contrast of the output beam is

$$C_{\downarrow}(\alpha) = \frac{(\epsilon - 1) |\cos(\frac{\alpha}{2})|}{\frac{\epsilon}{2}(1 + |\cos(\alpha)|) - 1} C_0 \quad (6)$$

where α is the angle of the spin-rotation. This is strictly less than the non-filtered phase contrast C_0 when the spin and path are coupled ($\alpha \neq 0$).

If we define a new quantity, the spin-contrast where our parameter of variation in intensity is the angle of spin rotation α , rather than the phase rotation ϕ , we get an expression for spin-contrast as follows:

$$C_{spin,\downarrow}(\phi) = \frac{\Omega(\phi)}{2 - \Omega(\phi)} \quad (7)$$

where $\Omega(\phi) = \epsilon + C_0(\epsilon - 1) |\cos(\phi)|$. Thus if the neutron spin is perfectly polarized, the spin-contrast is 1 regardless of the value for path contrast C_0 . However, if $\epsilon < 1$, the maximum spin-contrast is obtained when setting $\phi = 0$, and in this case the spin-contrast is greater than the original contrast C if the initial spin polarization satisfies $\epsilon \geq C_0/(1 + C_0)$.

In an actual experiment implementing the spin-rotation causes the variations in temperature of the interferometer which cause the phase to drift, increasing the effective strength of the phase noise in our interferometer. If we consider the case that the phase noise is sufficiently strong so that $C_0 \approx 0$, then the phase-averaged spin-contrast is given by

$$\overline{C}_{spin,\downarrow} = \frac{\epsilon}{2 - \epsilon} \quad (8)$$

which depends only on the initial polarization of the neutron beam.

To experimentally demonstrate the disturbance of the path state of the neutron beam due to measurements on

the spin state we compared the contrast of three neutron interferometers using the setup shown in Fig. 3. The experiment was performed at the National Institute of Standards and Technology Center for Neutron Research's Neutron Optics and Interferometer Facility, located at Gaithersburg, Maryland [18]. This facility has an excellent vibration isolation and temperature stability thus allows for a good and long phase stability [19]. Our neutron beam consisted of 0.271nm wavelength neutrons, and the incident neutron beam was polarized via a transmission-mode supermirror polarizer [20]. The path-selective spin-rotation was implemented using R. Pynn's thin permalloy films deposited on Si substrate [21]. As a spin-filters we have used a set of adiabatic coils (to rotate a neutron spin in the needed direction) coupled with either Heisler crystals or reflection-mode bended supermirrors. These were set to transmit spin-down neutrons. During this experimental work we have used two LLL type interferometers with different initial contrasts: "good" and "bad", which we refer to as N_1 and N_2 respectively. Also in order to simulate an interferometer with very low contrast we have used the "good" interferometer with 45 degree fussed silica wedge placed in one of NI paths which we refer to as N_3 [22].

The measured contrast curves without spin-filtering for the three neutron interferometers is shown in Fig. 4, these give contrast values of $C_1 = 82.5 \pm 1.3\%$, $C_2 = 23 \pm 1.5\%$, $C_3 = 2 \pm 1.7\%$ for the interferometers N_1, N_2 and N_3 respectively. These contrast values correspond to standard deviations of $\sigma_1 = 0.62 \pm 0.03$, $\sigma_2 = 1.71 \pm 0.04$, $\sigma_3 = 2.80 \pm 0.61$ respectively in the noise model under consideration.

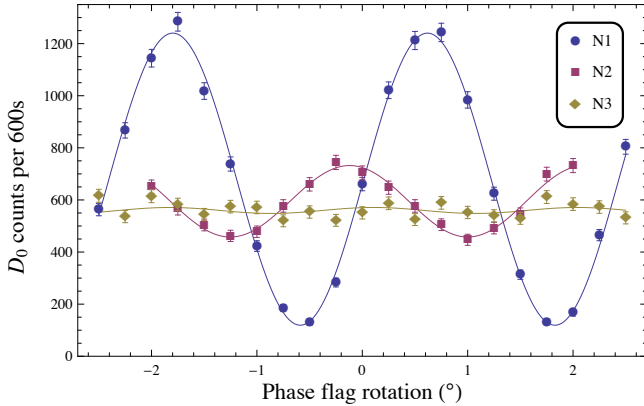


FIG. 4: Measured intensity curves at the D_0 as a function of phase-flag rotation for three neutron interferometers. The corresponding contrast values are $C_1 = 82.5 \pm 1.3\%$, $C_2 = 23 \pm 1.5\%$, $C_3 = 2 \pm 1.7\%$ for interferometers N_1, N_2, N_3 respectively.

After applying the spin-down filter, the spin-filtered contrasts were found to be $S_1 = 78.0 \pm 3\%$, $S_2 = 74.2 \pm 2.2\%$, $S_3 = 84 \pm 4\%$, as shown in Fig. 5. Our theoretical model with an initial neutron spin polarization of $(1 +$

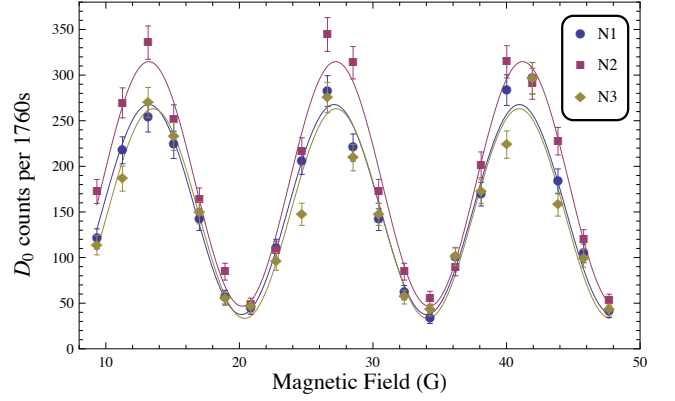


FIG. 5: Measured intensity curves at the D_0 as a function of spin-rotation for three neutron interferometers where we have applied a spin-filter on the output beam to select spin-down neutrons. The corresponding spin-filtered contrast values are $S_1 = 78.0 \pm 3\%$, $S_2 = 74.2 \pm 2.2\%$, $S_3 = 84 \pm 4\%$ for interferometers N_1, N_2, N_3 respectively.

$\epsilon)/2 = 93\%$ predicts an values of spin-contrast value of 75.3% for all three interferometers.

We have theoretically and experimentally investigated the role of quantum correlations in a simple bipartite quantum system in the presence of noise by using the spin and path degrees of freedom of a polarized neutron beam in a neutron interferometer. Our experimental results agree with our theoretical model predicting an increase in spin-filtered contrast over phase contrast for three interferometers. The deviations between our measured spin-filtered contrast the value predicted by our theoretical model are consistent with phase variations over the acquisition time due to temperature and humidity fluctuations in the neutron interferometer environment.

If we initially entangle the the path and spin systems of the neutron beam by a path dependent spin-rotation, in the presence of strong phase noise this entanglement is reduced to zero in the output beam state for all angles of the spin-rotation. However there are still non-classical correlations as quantified by a non-zero quantum discord $D(A|B)$ of the output beam. We interpreted the non-zero discord as a signature that even in the presence of strong phase noise, the neutron interferometer still exhibits genuine quantum behaviour, and therefore must still be treated as a quantum system.

This work was supported by the Canadian Excellence Research Chairs (CERC) program, the Canadian Institute for Advanced Research (CIFAR), and the Natural Sciences and Engineering Research Council of Canada (NSERC) Collaborative Research and Training Experience Program (CREATE). The authors gratefully acknowledge useful discussions with R. Pynn and G.-X. Miao, and use of permalloy films from M. Th. Rekvelde and R. Pynn.

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